Math 201, **Quiz II Long Sample 1** K. Aziziheris *(2013)*

1) Let $f(x,y) = \sqrt{9 - x^2 - y^2}$

b) From definitions, show that f_x (0, 0) = 0 = f_y (0, 0)

c) Prove that f is differentiable at (0, 0) d) sketch the graph of z = $\sqrt{9 - x^2 - y^2}$ (upper sphere)

a) Find the domain of f and describe the level curves of f.

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2) (i) Show that $\lim\limits_{(x,y \to (0,0)} \dfrac{xy^9}{x^2 + y^8} \cos(\dfrac{y}{x}) = 0.$ (ii) Let $\mathrm{f}(\mathrm{x}, \mathrm{y}) = 7\cos(\dfrac{\mathrm{x}^{4/3}\mathrm{y}^4}{\mathrm{x}^2 + \mathrm{y}^8})$ for $(\mathrm{x}, \mathrm{y}) \neq (0,0)$

Prove or disprove that $f(0,0)$ can be defined so that $f(x, y)$ is continous at (0, 0).

3) Suppose $F(x,y,z,w)=100$ and all components of ∇F are never zero.

Find $\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial x}$ given that $\frac{\partial x}{\partial z}=e^{3x-10y+7z}$. Justify your answer .

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4) The function $f(x,y,z)$ at a point P <u>increases</u> most rapidly in the direction of the vector v=(3,4,5) with directional derivative $10\sqrt{2}$. (i) Find $\nabla f(P)$

(ii) Find the directional derivative of $f(x,y,z)$ at P in the direction of the vector w=(4, 0, 3).

(iii) Is it possible to find a vector v such that D_v (f) (P) = 20 ? Explain.

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5) The derivative of of $f(x;\, y)$ at $P(1;\, 2)$ in the direction of i + j is $2\sqrt{2}$ and in direction of –2j is –3. Find the derivative of f in the direction of –i–2j.

(big Hint: Suppose grad(f)(P)= <a, b>. So you have 2 equations in 2 unknowns

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0) Find a, b if $f(x,y,z)=e^{ax+by}\cos 5z$ satisfies Laplace equation $f_{xx}+f_{yy}+f_{zz}=0$.

Baby 6) Given the surface $z = x^2 - 4xy + y^3 + 4y - 2$ containing the point $P(1; -1; -2)$

a) Find an equation of the tangent plane to the surface at P.

b) Find an equation of the normal line to the surface at P.

7a) Investigate the critical points of

$f(x, y) = 2x^3 + 6xy + 2y^3 + 17$ for local maxima, local minima, or saddle points.

7b) Locate all local extrema and saddle points of $f(x, y) = x^3 - y^3 - 2xy + 6$

7c) Locate all local extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

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8) Find the parametric equations for the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + y^2 + z^2 = 6$ at the point $P(1; 1; 1)$. (big Hint: cross products).

9) By about how much will $f(x; y; z) = \ln\sqrt{x^2 + y^2 + z^2}$ change if the point $p(x; y; z)$ moves from P_0 (3; 4; 12) a distance of 0.1 units in the direction of 3i + 6j − 2k ?

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10) Find the set of points <u>on the surface</u> $x^2 + y^2 - 36 = 8xyz$ where the tangent plane is

(i) perpendicular to the x-y plane. (ii) parallel to the x-y plane.

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11) (14.8) Use Lagrange multipliers to find the (absolute) maximum and minimum of the function $f(x, y, z) = 5x - 2y + z + 17$ on the surface $x^2 + y^2 + z^2 = 30$

12) (Chain Rule) Suppose $\nabla f(1,1,1) = \mathbf{5i} + 3\mathbf{j} + 4\mathbf{k}$ and $f(1,1,1)=5$.

Let $P = f(t^4, t^2, tx^2)$ where $f(u,v,w)$ is a differentiable function. Then at t=1, x=1,

(i) $\partial P / \partial t = \ldots\ldots$ (ii) $\partial(x^3 P) / \partial t = \ldots.$ (iii) $\partial(t^3 P) / \partial t = \ldots\ldots.$

12*) Suppose $f(tx, ty) = t^5 f(x, y)$ for all values of x, y, t (where $f(u,v)$ is a differentiable function). Show that (i) $xf_x + yf_y = 5f$ (Hint: Partial w.r.t t both sides, then set t=1).

(ii) $x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy} = 20f$ (Hint: Double partial w.r.t t both sides, then set t=1).

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13) Find the absolute minimum and maximum of the function $f(x,y) = x^2 + 2y^2 - y - 1$

a) Over the region $R = \{(x,y): \quad x^2 + y^2 \leq 1\}$

b) Over the region $R = \{(x,y): \quad x^2 + y^2 \leq 1 \text{ and } y \geq 0\}$

Good Luck